Таблица связи тригонометрических функций

	sin x	cos x	tg x	ctg x
sin x		$\pm\sqrt{1-\cos^2 x}$	$\dfrac{tg\ x}{\pm\sqrt{1+tg^2 x}}$	$\dfrac{1}{\pm\sqrt{1+ctg^2 x}}$
cos x	$\pm\sqrt{1-\sin^2 x}$		$\dfrac{1}{\pm\sqrt{1+tg^2 x}}$	$\dfrac{ctg\ x}{\pm\sqrt{1+ctg^2 x}}$
tg x	$\dfrac{\sin x}{\pm\sqrt{1-\sin^2 x}}$	$\dfrac{\pm\sqrt{1-\cos^2 x}}{\cos x}$		$\dfrac{1}{ctg\ x}$
ctg x	$\dfrac{\pm\sqrt{1-\sin^2 x}}{\sin x}$	$\dfrac{\cos x}{\pm\sqrt{1-\cos^2 x}}$	$\dfrac{1}{tg\ x}$	

$$\sin^2 x + \cos^2 x = 1 \quad \forall x \in \mathbb{R} \tag{1}$$

$$tg^2 x + 1 = \frac{1}{\cos^2 x} \quad \forall x \in \mathbb{R} \setminus \left\{\frac{\pi}{2} + \pi k\right\}, k \in \mathbb{Z} \tag{2}$$

$$ctg^2 x + 1 = \frac{1}{\sin^2 x} \quad \forall x \in \mathbb{R} \setminus \left\{\pi k\right\}, k \in \mathbb{Z} \tag{3}$$

$$tg\ x \cdot ctg\ x = 1 \quad \forall x \in \mathbb{R} \setminus \left\{\frac{\pi k}{2}\right\}, k \in \mathbb{Z} \tag{4}$$

I. Синус

1. *из* $(1) \Rightarrow \sin^2 x = 1 - \cos^2 x \Leftrightarrow \sin x = \pm\sqrt{1-\cos^2 x}$;

2. *из* $(1) \Rightarrow \cos^2 x = 1 - \sin^2 x$;

из $(2) \Rightarrow (tg^2 x + 1)\cos^2 x = 1 \Leftrightarrow (tg^2 x + 1)(1 - \sin^2 x) = 1 \Leftrightarrow$

$\Leftrightarrow tg^2 x + 1 - (tg^2 x + 1)\sin^2 x = 1 \Leftrightarrow tg^2 x = (1 + tg^2 x)\sin^2 x \Leftrightarrow$

$\Leftrightarrow \sin^2 x = \dfrac{tg^2 x}{1 + tg^2 x} \Leftrightarrow \sin^2 x - \dfrac{tg^2 x}{1 + tg^2 x} = 0 \Leftrightarrow$

$\Leftrightarrow \left(\sin x - \dfrac{tg\ x}{\sqrt{1+tg^2 x}}\right)\left(\sin x + \dfrac{tg\ x}{\sqrt{1+tg^2 x}}\right) = 0 \Leftrightarrow \sin x = \dfrac{tg\ x}{\pm\sqrt{1+tg^2 x}}$;

3. $uз$ $(3) \Rightarrow (ctg^2x+1)\sin^2 x = 1 \Leftrightarrow \sin^2 x = \dfrac{1}{1+ctg^2x} \Leftrightarrow \sin^2 x - \dfrac{1}{1+ctg^2x} = 0 \Leftrightarrow$

$$\Leftrightarrow \left(\sin x - \dfrac{1}{\sqrt{1+ctg^2x}}\right)\left(\sin x + \dfrac{1}{\sqrt{1+ctg^2x}}\right) = 0 \Leftrightarrow \sin x = \dfrac{1}{\pm\sqrt{1+ctg^2x}}.$$

II. Косинус

1. $uз$ $(1) \Rightarrow \cos^2 x = 1 - \sin^2 x \Leftrightarrow \cos x = \pm\sqrt{1-\sin^2 x};$

2. $uз$ $(2) \Rightarrow (tg^2x+1)\cos^2 x = 1 \Leftrightarrow \cos^2 x = \dfrac{1}{1+tg^2x} \Leftrightarrow \cos^2 x - \dfrac{1}{1+tg^2x} = 0 \Leftrightarrow$

$$\Leftrightarrow \left(\cos x - \dfrac{1}{\sqrt{1+tg^2x}}\right)\left(\cos x + \dfrac{1}{\sqrt{1+tg^2x}}\right) = 0 \Leftrightarrow \cos x = \dfrac{1}{\pm\sqrt{1+tg^2x}};$$

3. $uз$ $(1) \Rightarrow \sin^2 x = 1 - \cos^2 x;$

$uз$ $(3) \Rightarrow (ctg^2x+1)\sin^2 x = 1 \Leftrightarrow (ctg^2x+1)(1-\cos^2 x) = 1 \Leftrightarrow$

$\Leftrightarrow ctg^2x + 1 - (ctg^2x+1)\cos^2 x = 1 \Leftrightarrow ctg^2x = (1+ctg^2x)\cos^2 x \Leftrightarrow$

$\Leftrightarrow \cos^2 x = \dfrac{ctg^2x}{1+ctg^2x} \Leftrightarrow \cos^2 x - \dfrac{ctg^2x}{1+ctg^2x} = 0 \Leftrightarrow$

$$\Leftrightarrow \left(\cos x - \dfrac{ctg\,x}{\sqrt{1+ctg^2x}}\right)\left(\cos x + \dfrac{ctg\,x}{\sqrt{1+ctg^2x}}\right) = 0 \Leftrightarrow \cos x = \dfrac{ctg\,x}{\pm\sqrt{1+ctg^2x}}.$$

III. Тангенс

1. $uз$ $(1) \Rightarrow \cos^2 x = 1 - \sin^2 x;$

$tg\,x \overset{def}{=} \dfrac{\sin x}{\cos x} \Rightarrow tg^2x = \dfrac{\sin^2 x}{\cos^2 x} \Leftrightarrow tg^2x = \dfrac{\sin^2 x}{1-\sin^2 x} \Leftrightarrow tg^2x - \dfrac{\sin^2 x}{1-\sin^2 x} = 0 \Leftrightarrow$

$$\Leftrightarrow \left(tg\,x - \dfrac{\sin x}{\sqrt{1-\sin^2 x}}\right)\left(tg\,x + \dfrac{\sin x}{\sqrt{1-\sin^2 x}}\right) = 0 \Leftrightarrow tg\,x = \dfrac{\sin x}{\pm\sqrt{1-\sin^2 x}};$$

2. $uз$ $(2) \Rightarrow tg^2x = \dfrac{1-\cos^2 x}{\cos^2 x} \Leftrightarrow tg^2x - \dfrac{1-\cos^2 x}{\cos^2 x} = 0 \Leftrightarrow$

$$\Leftrightarrow \left(tg\,x - \dfrac{\sqrt{1-\cos^2 x}}{\cos x}\right)\left(tg\,x + \dfrac{\sqrt{1-\cos^2 x}}{\cos x}\right) = 0 \Leftrightarrow tg\,x = \dfrac{\pm\sqrt{1-\cos^2 x}}{\cos x};$$

3. *из* $(4) \Rightarrow tg\ x = \dfrac{1}{ctg\ x}.$

IV. Котангенс

1. *из* $(3) \Rightarrow ctg^2 x = \dfrac{1 - \sin^2 x}{\sin^2 x} \Leftrightarrow ctg^2 x - \dfrac{1 - \sin^2 x}{\sin^2 x} = 0 \Leftrightarrow$

$$\Leftrightarrow \left(ctg\ x - \dfrac{\sqrt{1 - \sin^2 x}}{\sin x} \right) \left(ctg\ x + \dfrac{\sqrt{1 - \sin^2 x}}{\sin x} \right) = 0 \Leftrightarrow ctg\ x = \dfrac{\pm\sqrt{1 - \sin^2 x}}{\sin x};$$

2. *из* $(1) \Rightarrow \sin^2 x = 1 - \cos^2 x;$

$$ctg\ x \overset{def}{=} \dfrac{\cos x}{\sin x} \Rightarrow ctg^2 x = \dfrac{\cos^2 x}{\sin^2 x} \Leftrightarrow ctg^2 x = \dfrac{\cos^2 x}{1 - \cos^2 x} \Leftrightarrow ctg^2 x - \dfrac{\cos^2 x}{1 - \cos^2 x} = 0 \Leftrightarrow$$

$$\Leftrightarrow \left(ctg\ x - \dfrac{\cos x}{\sqrt{1 - \cos^2 x}} \right) \left(ctg\ x + \dfrac{\cos x}{\sqrt{1 - \cos^2 x}} \right) = 0 \Leftrightarrow ctg\ x = \dfrac{\cos x}{\pm\sqrt{1 - \cos^2 x}};$$

3. *из* $(4) \Rightarrow ctg\ x = \dfrac{1}{tg\ x}.$