Таблица связи обратных тригонометрических функций

	arcsin y	***arccos y***	***arctg y***	***arcctg y***
arcsin x		$y=\sqrt{1-x^2}$ $0\le x\le 1$	$y=\dfrac{x}{\sqrt{1-x^2}}$ $0<x<1$	$y=\dfrac{\sqrt{1-x^2}}{x}$ $0<x\le 1$
arccos x	$y=\sqrt{1-x^2}$ $0\le x\le 1$		$y=\dfrac{\sqrt{1-x^2}}{x}$ $0<x\le 1$	$y=\dfrac{x}{\sqrt{1-x^2}}$ $0<x<1$
arctg x	$y=\dfrac{x}{\sqrt{1+x^2}}$ $x\in\mathbb{R}$	$y=\dfrac{1}{\sqrt{1+x^2}}$ $x\ge 0$		$y=\dfrac{1}{x}$ $x>0$
arccg x	$y=\dfrac{1}{\sqrt{1+x^2}}$ $x\ge 0$	$y=\dfrac{x}{\sqrt{1+x^2}}$ $x\in\mathbb{R}$	$y=\dfrac{1}{x}$ $x>0$	

<u>I. Арксинус</u>

1. $\arcsin x=\arccos y=t;$

$$\begin{cases}-\dfrac{\pi}{2}\le\arcsin x\le\dfrac{\pi}{2},\\ 0\le\arccos y\le\pi;\end{cases}\Rightarrow 0\le t\le\frac{\pi}{2}\Leftrightarrow\begin{cases}0\le\arcsin x\le\dfrac{\pi}{2},\\ 0\le\arccos y\le\dfrac{\pi}{2};\end{cases}\Leftrightarrow\begin{cases}0\le x\le 1,\\ 0\le y\le 1.\end{cases};$$

$$\begin{cases}\arcsin x=t,\\ \arccos y=t;\end{cases}\Leftrightarrow\begin{cases}x=\sin t,\\ y=\cos t.\end{cases};\ 0\le t\le\frac{\pi}{2}\Rightarrow\cos t=\sqrt{1-\sin^2 t}\Rightarrow y=\sqrt{1-x^2}.$$

2. $\arcsin x = \operatorname{arc}tg\ y = t;$

$$\begin{cases} -\frac{\pi}{2} \le \arcsin x \le \frac{\pi}{2}, \\ -\frac{\pi}{2} < \operatorname{arc}tg\ y < \frac{\pi}{2}; \end{cases} \Rightarrow -\frac{\pi}{2} < t < \frac{\pi}{2} \Leftrightarrow \begin{cases} -\frac{\pi}{2} < \arcsin x < \frac{\pi}{2}, \\ -\frac{\pi}{2} < \operatorname{arc}tg\ y < \frac{\pi}{2}; \end{cases} \Leftrightarrow \begin{cases} 0 < x < 1, \\ y \in \mathbb{R}. \end{cases};$$

$$\begin{cases} \arcsin x = t, \\ \operatorname{arc}tg\ y = t; \end{cases} \Leftrightarrow \begin{cases} x = \sin t, \\ y = tg\ t. \end{cases}; \ -\frac{\pi}{2} < t < \frac{\pi}{2} \Rightarrow tg\ t = \frac{\sin t}{\sqrt{1-\sin^2 t}} \Rightarrow y = \frac{x}{\sqrt{1-x^2}}.$$

3. $\arcsin x = \operatorname{arc}ctg\ y = t;$

$$\begin{cases} -\frac{\pi}{2} \le \arcsin x \le \frac{\pi}{2}, \\ 0 < \operatorname{arc}ctg\ y < \pi; \end{cases} \Rightarrow 0 < t \le \frac{\pi}{2} \Leftrightarrow \begin{cases} 0 < \arcsin x \le \frac{\pi}{2}, \\ 0 < \operatorname{arc}tg\ y \le \frac{\pi}{2}; \end{cases} \Leftrightarrow \begin{cases} 0 < x \le 1, \\ y \ge 0. \end{cases};$$

$$\begin{cases} \arcsin x = t, \\ \operatorname{arc}ctg\ y = t; \end{cases} \Leftrightarrow \begin{cases} x = \sin t, \\ y = ctg\ t. \end{cases}; \ 0 < t \le \frac{\pi}{2} \Rightarrow ctg\ t = \frac{\sqrt{1-\sin^2 t}}{\sin t} \Rightarrow y = \frac{\sqrt{1-x^2}}{x}.$$

II. Арккосинус

1. $\arccos x = \arcsin y = t;$

$$\begin{cases} 0 \le \arccos x \le \pi, \\ -\frac{\pi}{2} \le \arcsin y \le \frac{\pi}{2}; \end{cases} \Rightarrow 0 \le t \le \frac{\pi}{2} \Leftrightarrow \begin{cases} 0 \le \arccos x \le \frac{\pi}{2}, \\ 0 \le \arcsin y \le \frac{\pi}{2}; \end{cases} \Leftrightarrow \begin{cases} 0 \le x \le 1, \\ 0 \le y \le 1. \end{cases};$$

$$\begin{cases} \arccos x = t, \\ \arcsin y = t; \end{cases} \Leftrightarrow \begin{cases} x = \cos t, \\ y = \sin t. \end{cases}; \ 0 \le t \le \frac{\pi}{2} \Rightarrow \sin t = \sqrt{1-\cos^2 t} \Rightarrow y = \sqrt{1-x^2}.$$

2. $\arccos x = \operatorname{arc}tg\ \ y = t;$

$$\begin{cases} 0 \le \arccos x \le \pi, \\ -\dfrac{\pi}{2} < \operatorname{arc}tg\ \ y < \dfrac{\pi}{2}; \end{cases} \Rightarrow 0 \le t < \frac{\pi}{2} \Leftrightarrow \begin{cases} 0 \le \arccos x < \dfrac{\pi}{2}, \\ 0 \le \operatorname{arc}tg\ \ y < \dfrac{\pi}{2}; \end{cases} \Leftrightarrow \begin{cases} 0 < x \le 1, \\ y \ge 0. \end{cases};$$

$$\begin{cases} \arccos x = t, \\ \operatorname{arc}tg\ \ y = t; \end{cases} \Leftrightarrow \begin{cases} x = \cos t, \\ y = tg\ \ y. \end{cases};\ \ 0 \le t < \frac{\pi}{2} \Rightarrow tg\ \ t = \frac{\sqrt{1-\cos^2 t}}{\cos t} \Rightarrow y = \frac{\sqrt{1-x^2}}{x}.$$

3. $\arccos x = \operatorname{arc}ctg\ \ y = t;$

$$\begin{cases} 0 \le \arccos x \le \pi, \\ 0 < \operatorname{arc}ctg\ \ y < \pi; \end{cases} \Rightarrow 0 < t < \pi \Leftrightarrow \begin{cases} 0 < \arccos x < \pi, \\ 0 < \operatorname{arc}ctg\ \ y < \pi; \end{cases} \Leftrightarrow \begin{cases} -1 < x < 1, \\ y \in \mathbb{R}. \end{cases};$$

$$\begin{cases} \arccos x = t, \\ \operatorname{arc}ctg\ \ y = t; \end{cases} \Leftrightarrow \begin{cases} x = \cos t, \\ y = ctg\ \ y. \end{cases};\ \ 0 < t < \pi \Rightarrow ctg\ \ t = \frac{\cos t}{\sqrt{1-\cos^2 t}} \Rightarrow y = \frac{x}{\sqrt{1-x^2}}.$$

III. Арктангенс

1. $\operatorname{arc}tg\ \ x = \arcsin y = t;$

$$\begin{cases} -\dfrac{\pi}{2} < \operatorname{arc}tg\ \ x < \dfrac{\pi}{2}, \\ -\dfrac{\pi}{2} \le \arcsin y \le \dfrac{\pi}{2}; \end{cases} \Rightarrow -\frac{\pi}{2} < t < \frac{\pi}{2} \Leftrightarrow \begin{cases} -\dfrac{\pi}{2} < \operatorname{arc}tg\ \ x < \dfrac{\pi}{2}, \\ -\dfrac{\pi}{2} < \arcsin y < \dfrac{\pi}{2}; \end{cases} \Leftrightarrow \begin{cases} x \in \mathbb{R}, \\ -1 < y < 1. \end{cases};$$

$$\begin{cases} \operatorname{arc}tg\ \ x = t, \\ \arcsin y = t; \end{cases} \Leftrightarrow \begin{cases} x = tg\ \ t, \\ y = \sin t. \end{cases};\ \ -\frac{\pi}{2} < t < \frac{\pi}{2} \Rightarrow \sin t = \frac{tg\ \ t}{\sqrt{1+tg^2 t}} \Rightarrow y = \frac{x}{\sqrt{1+x^2}}.$$

2. $\operatorname{arc}tg\ \ x = \arccos y = t;$

$$\begin{cases} -\dfrac{\pi}{2} < \operatorname{arc}tg\ \ x < \dfrac{\pi}{2}, \\ 0 \le \arccos y \le \pi; \end{cases} \Rightarrow 0 \le t < \frac{\pi}{2} \Leftrightarrow \begin{cases} 0 \le \operatorname{arc}tg\ \ x < \dfrac{\pi}{2}, \\ 0 \le \arccos y < \dfrac{\pi}{2}; \end{cases} \Leftrightarrow \begin{cases} x \ge 0, \\ 0 < y \le 1. \end{cases};$$

$$\begin{cases} \operatorname{arc}tg\ \ x = t, \\ \arccos y = t; \end{cases} \Leftrightarrow \begin{cases} x = tg\ \ t, \\ y = \cos t. \end{cases};\ \ 0 \le t < \frac{\pi}{2} \Rightarrow \cos t = \frac{1}{\sqrt{1+tg^2 t}} \Rightarrow y = \frac{1}{\sqrt{1+x^2}}.$$

3. $\operatorname{arc}tg\ x = \operatorname{arc}ctg\ y = t;$

$$\begin{cases} -\dfrac{\pi}{2} < \operatorname{arc}tg\ x < \dfrac{\pi}{2}, \\ 0 < \operatorname{arc}ctg\ y < \pi; \end{cases} \Rightarrow 0 < t < \frac{\pi}{2} \Leftrightarrow \begin{cases} 0 < \operatorname{arc}tg\ x < \dfrac{\pi}{2}, \\ 0 < \operatorname{arc}ctg\ y < \dfrac{\pi}{2}; \end{cases} \Leftrightarrow \begin{cases} x > 0, \\ y > 0. \end{cases};$$

$$\begin{cases} \operatorname{arc}tg\ x = t, \\ \operatorname{arc}ctg\ y = t; \end{cases} \Leftrightarrow \begin{cases} x = tg\ t, \\ y = ctg\ t. \end{cases};\ ctg\ t = \frac{1}{tg\ t} \Rightarrow y = \frac{1}{x}.$$

IV. Арккотангенс

1. $\operatorname{arc}ctg\ x = \arcsin y = t;$

$$\begin{cases} 0 < \operatorname{arc}ctg\ x < \pi, \\ -\dfrac{\pi}{2} \le \arcsin y \le \dfrac{\pi}{2}; \end{cases} \Rightarrow 0 < t \le \frac{\pi}{2} \Leftrightarrow \begin{cases} 0 < \operatorname{arc}ctg\ x \le \dfrac{\pi}{2}, \\ 0 < \arcsin y \le \dfrac{\pi}{2}; \end{cases} \Leftrightarrow \begin{cases} x \ge 0, \\ 0 < y \le 1. \end{cases};$$

$$\begin{cases} \operatorname{arc}ctg\ x = t, \\ \arcsin y = t; \end{cases} \Leftrightarrow \begin{cases} x = ctg\ t, \\ y = \sin t. \end{cases};\ 0 < t \le \frac{\pi}{2} \Rightarrow \sin t = \frac{1}{\sqrt{1 + ctg^2 t}} \Rightarrow y = \frac{1}{\sqrt{1 + x^2}}.$$

2. $\operatorname{arc}ctg\ x = \arccos y = t;$

$$\begin{cases} 0 < \operatorname{arc}ctg\ x < \pi, \\ 0 \le \arccos y \le \pi; \end{cases} \Rightarrow 0 < t < \pi \Leftrightarrow \begin{cases} 0 < \operatorname{arc}ctg\ x < \pi, \\ 0 < \arccos y < \pi; \end{cases} \Leftrightarrow \begin{cases} x \in \mathbb{R}, \\ -1 < y < 1. \end{cases};$$

$$\begin{cases} \operatorname{arc}ctg\ x = t, \\ \arccos y = t; \end{cases} \Leftrightarrow \begin{cases} x = ctg\ t, \\ y = \cos t. \end{cases};\ 0 < t < \pi \Rightarrow \cos t = \frac{ctg\ t}{\sqrt{1 + ctg^2 t}} \Rightarrow y = \frac{x}{\sqrt{1 + x^2}}.$$

3. $\operatorname{arc}ctg\ x = \operatorname{arc}tg\ y = t;$

$$\begin{cases} 0 < \operatorname{arc}ctg\ x < \pi, \\ -\dfrac{\pi}{2} < \operatorname{arc}tg\ y < \dfrac{\pi}{2}; \end{cases} \Rightarrow 0 < t < \frac{\pi}{2} \Leftrightarrow \begin{cases} 0 < \operatorname{arc}ctg\ x < \dfrac{\pi}{2}, \\ 0 < \operatorname{arc}tg\ y < \dfrac{\pi}{2}; \end{cases} \Leftrightarrow \begin{cases} x > 0, \\ y > 0. \end{cases};$$

$$\begin{cases} \operatorname{arc}ctg\ x = t, \\ \operatorname{arc}tg\ y = t; \end{cases} \Leftrightarrow \begin{cases} x = ctg\ t, \\ y = tg\ t. \end{cases};\ tg\ t = \frac{1}{ctg\ t} \Rightarrow y = \frac{1}{x}.$$